HEAT TRANSFER IN THE NEIGHBORHOOD OF THE STAGNATION POINT WITH DISCONTINUOUS PRANDTL NUMBERS

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The transport properties of real gases vary very sharply under conditions of dissociation and ionization. In order to analyze the effect of these variations on heat transfer, it is worthwhile considering a model with discontinuous transport properties. In this note we shall consider the effect of discontinuous Prandtl numbers on heat transfer.

Lees [1] has shown that in the case of a strongly cooled wall, with constant product of viscosity and density across the boundary layer, the velocity and enthalpy of the gas are determined by the system (in the usual notation):

$$2f''' + ff'' = 0, \quad (i'/\sigma)' + {}^1/_2 fi' = 0\,, \qquad f = f' = 0, \quad i = i_w \quad \text{for} \quad \eta = 0\,, \quad f' \to 1, \quad i \to 1 \quad \text{for} \quad \eta \to \infty\,. \tag{1}$$

When the Prandtl number is discontinuous at a point $\eta = \eta_0$, the enthalpy and heat flux at that point must satisfy the continuity conditions:

$$i(\eta_0 - 0) = i(\eta_0 + 0), \qquad \sigma_2 i'(\eta_0 - 0) = \sigma_1 i'(\eta_0 + 0)\,, \qquad \sigma = \sigma_1 \quad \text{for} \quad \eta \leqslant \eta_0, \qquad \sigma = \sigma_2 \quad \text{for} \quad \eta > \eta_0\,. \tag{2}$$

Then, using (1) and (2), we can easily obtain by quadratures the value of $i'(0)$ determining the heat transfer to the wall surface.

We are primarily interested in two cases:

1. The Prandtl number is equal to unity (the Prandtl number is assumed to be $\sigma = 1$ purely for simplicity) everywhere except for a narrow range $\eta_1 \leq \eta \leq \eta_2$, where $\sigma = \sigma_1 =$ const. We shall assume that the range $\Delta\eta = \eta_2 - \eta_1$ is so small that terms $O(\Delta\eta^2)$ may be neglected. Then it is easy to show that

$$i'(0) = (1 - i_w) f''(0) \left\{ f'(\eta_1) + \frac{2f''(\eta_1)}{f(\eta_1)} \left(1 - \exp\frac{-\sigma f(\eta_1)\,\Delta\eta}{2}\right) + \exp\frac{-\sigma f(\eta_1)\,\Delta\eta}{2} \left[1 - f'(\eta_1) - \Delta\eta f''(\eta_1) + \Delta\eta \frac{f(\eta_1)}{2}(1 - f'(\eta_1))\right]\right\}^{-1} + O(\Delta\eta^2)\,. \tag{3}$$

Hence it follows that if the gas has an infinitely high thermal conductivity in the interval $\Delta\eta$,

$$i'(0) = (1 - i_w) f''(0) \qquad \text{for} \quad \sigma_1 \Delta\eta \to 0\,. \tag{4}$$

This is the same value as when $\sigma = 1$ throughout the boundary layer. Thus a thin layer of gas with high thermal conductivity does not affect the heat flux at the wall.

The situation is different for a layer of gas with low thermal conductivity. The degree to which such a layer can affect the heat flux to the wall depends strongly on its position. At high η it has practically no effect on the heat flux, whereas when it is near the wall its effect may be arbitrarily strong. The physical explanation is that the layer behaves like a filter, which permits heat transfer associated with a mass flux, but impedes conductive heat transfer. And since the mass flux varies across the boundary layer, the heat flux to the wall must vary accordingly. In the general case, with prescribed $\sigma_1\Delta\eta$, the wall heat flux is determined by (3).

In heat-transfer calculations one often uses the notion of a characteristic enthalpy (or temperature), which determines the gas properties, including the Prandtl number σ. The characteristic enthalpy usually depends linearly on the wall temperature: Thus, when the wall temperature changes, the wall enthalpy will pass through a value i_w for which the heat flux, as determined by the characteristic-enthalpy method, will experience a discontinuity, whereas, according to (3), the heat flux is a continuous function of i_w. This indicates that the characteristic-enthalpy method does not lead to satisfactory results in the case of discontinuous (or strongly varying) gas properties. The good agreement between the results of heat-flux calculations by the characteristic-enthalpy method and experimental data is because most experimental data have been obtained for relatively low stagnation temperatures, for which the variation of the gas properties across the boundary layer satisfies "similarity" conditions (the Prandtl number is practically constant, and the product of density and viscosity is a power function of the enthalpy). These "similarity" properties also enable us to obtain satisfactory formulas for the heat flux at relatively low temperatures by the characteristic-enthalpy method.

2. Fig. 1 shows the Prandtl number as a function of enthalpy for a dissociated gas at equilibrium when p = 1 atm (the curves corresponding to other pressures are similar). It is required to calculate the wall heat flux for the given dependence of the Prandtl number on enthalpy.

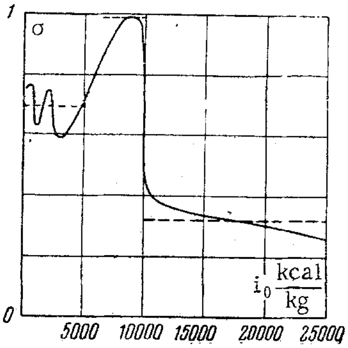

Fig. 1.

The broken line in Fig. 1 represents a step function which will serve as a first approximation to this dependence. In accordance with Fig. 1, we shall assume that the Prandtl numbers are prescribed as follows:

$$\eta \leqslant \eta_1, \quad \sigma = \sigma_1; \qquad \eta_1 < \eta < \eta_2, \quad \sigma = 1; \qquad \eta \geqslant \eta_2, \quad \sigma = \sigma_2 .$$

In this case $i'(0)$ is given by

$$i'(0) = (1 - i_w)\,[f''(0)]^{\sigma_1} \left\{ \int_0^{\eta_1} [f''(\eta)]^{\sigma_1} d\eta + \frac{1}{\sigma_1} \frac{f'(\eta_2) - f'(\eta_1)}{[f''(\eta_1)]^{1-\sigma_1}} + \frac{\sigma_2}{\sigma_1} \frac{[f''(\eta_2)]^{1-\sigma_2}}{[f''(\eta_1)]^{1-\sigma_1}} \int_{\eta_2}^{\infty} [f''(\eta)]^{\sigma_2} d\eta \right\}^{-1}. \tag{5}$$

The values of the enthalpy at the points of discontinuity can be calculated from

$$i(\eta_1) = i_w + i'(0) \int_0^{\eta_1} \left[\frac{f''(\eta)}{f''(0)} \right]^{\sigma_1} d\eta, \tag{6}$$

$$i(\eta_2) = i(\eta_1) + \frac{i'(0)}{\sigma_1} \left[\frac{f''(\eta_1)}{f''(0)} \right]^{\sigma_1} \frac{f'(\eta_2) - f'(\eta_1)}{f''(\eta_1)} . \tag{7}$$

In accordance with Fig. 1, we can assume $\sigma_1 = 0.7$, $\sigma_2 = 0.32$ for the enthalpy range $i_0 \leqslant 25\,000$ kcal/kg. These values were used in Fig. 2, which shows $i'(0)$ as a function of η_1, η_2, with $I = i'(0)/(1 - i_w)$.

When the enthalpy at the outer edge of the boundary layer is $i_e \sim 25\,000$ kcal/kg, Fig. 2 shows that for $i_w \ll 1$ the wall heat flux is 30% higher for the enthalpy dependence of the Prandtl numbers given by Fig. 1 than for a constant value $\sigma = 0.7$ throughout the boundary layer.

Note that in physically meaningful problems the values of $i(\eta_1)$, $i(\eta_2)$ are usually prescribed. In these cases $i'(0)$, η_1, η_2 can be determined from the system (5)-(7).

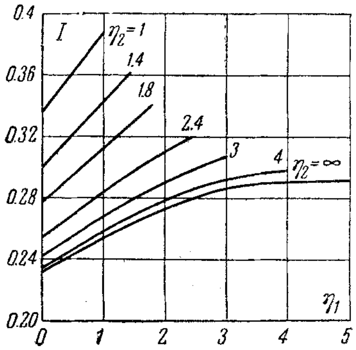

Fig. 2

REFERENCES

1. L. Lees, "Laminar heat transfer over blunt-nosed bodies at hypersonic flight speeds," Jet Propuls., no. 4, 1956.

2. C. F. Hansen, "Approximations for thermodynamic and transport properties of high-temperature air," NASA Technical Report R-50, 1959.

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